Mathematics 1220 COMPUTER EXERCISES Spring 2009

Complete the following exercises using Maple except where otherwise indicated, where you will need to do it by hand. **The assignment is due Wednesday, April 29 at 1pm. You can give the assignments to the instructor on/before Wednesday's class or to Paula Tooman in the math office (JWB 233)**. Although we understand you will experience the final "crunch" as the semester ends, we ask that there are as few conflicts as possible with the due date for these computer exercises. From our perspective, we have to coordinate grading of the final exams and computer assignments, as well as finalize grades within a few days of the end of the semester for over 250 students.

Submission Guidelines:

- Please print out all **relevant** Maple code and output. Read another way, this means that we ask that you NOT to hand in random Maple code/output in an attempt to get partial marks. If you're unable to solve a question but still seek partial credit, it's better to provide some code snippets summarizing the approach(es) you used to solve a problem and, perhaps, some explanation as to why you used that approach or what you were hoping to get. Only provide representative output if you feel it's relevant to explaining why you couldn't proceed further with the solution.

- Related to the above but worthy of special mention, if you have a long `for`... `end` loop, please suppress the output. You can display the relevant intermediate/final result of the for loop outside of the for loop if you feel it's necessary to show it.

- Some Maple functions/subjects that may be of interest are the following:

restart	taylor	plot	plot/details	plot/options	int	solve
infinity	piecewise	dsolve	diff	sin	exp	for

  To get help on any functions/subject, you can press Ctrl + F1 or, on a Maple command line, type `> ?X` where `X` is a function/subject. For example, `> ?plot` will show you help on the plot function, while `> ?plot/options` will tell you about the optional arguments that go in the plot function. For more help, explore the "Help" menu and then ask a TA.

- The TA's can help you with this assignment, but we recommend that you show us your code when you're asking a troubleshooting question (i.e. "Why doesn't the plot function plot my functions?"). Therefore, print or e-mail the Maple worksheet with your code, or bring your computer to show us what you've done. You can solve the problems below using a variety of approaches and it's difficult for us to answer your questions without first seeing the approach you're using.

- Use `shift` + `enter` to start a new line in the same execution block. If you just press `enter` after a line of code, then that will start a new execution block. Placing code in the same execution block will allow you to execute a (large) piece of code without having to press enter for each line of code.

- Because some of the questions on this assignment require you to use Maple and show some "by hand" analytical work, please package your assignment in a way that is easy for the graders to follow. You are not required to typeset your analytical work in Maple.
- You are strongly encouraged to comment your code (see the Insert → Text menu) to help the graders follow your work.
- Start this assignment sooner than when you think you need to, especially if you have no/little experience working with Maple!

Maple Questions

1. (a) Find the Taylor series expansion to order 10 of $f(x) = 1/(1-x)$ around $x = 0$.
   (b) Plot the graphs of $1/(1-x)$ and the Taylor polynomials $P_1(x), P_2(x)$, and $P_3(x)$ on the same axes. Display these graphs on an appropriate scale and include a legend on your plots to indicate what curves correspond to what Taylor polynomials.
   (c) Do the above again but for $g(x) = \sin x$, and plot the simultaneous graphs of $\sin x, P_1(x), P_3(x)$, and $P_5(x)$.

2. (a) Use the trapezoidal rule with various values for n to approximate $I = \int_0^1 x^3\, dx$. You should demonstrate how the accuracy improves as n increases.
   (b) Find the smallest n such that the approximation I_n is accurate to within four decimal places (that is, the smallest n such that if I_n is rounded off to four decimal places, then the result is equal to the actual value $I = 1/4$).
   (c) Find an accurate numerical value for $\int_{-\infty}^{\infty} e^{-x^2/2}\, dx$.

3. Let $f(x) = \begin{cases} -1, & -\pi < x < 0 \\ +1, & 0 < x < \pi \end{cases}$. Let $\mathcal{F}_N$ be the N^{th} Fourier approximation to $f(x)$, given by

$$\mathcal{F}_N(x) = \sum_{n=1}^{N} a_n \sin(nx), \qquad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\, dx.$$

   Calculate a_1 through a_{10} using Maple, and then make three separate graphs of $f(x)$ and $\mathcal{F}_N(x)$ on the same axes for $N = 1, 3, 5$. Check your results by calculating a_n analytically.

4. Solve the following using Maple and confirm the result analytically (i.e. by hand).

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = x \qquad y(0) = \frac{1}{2}, y'(0) = 0$$