

Mathematics 1220 PRACTICE EXAM I Spring 2009

1. Calculate the following limits.

(a) $\lim_{n \rightarrow \infty} \left(\frac{n-1}{n}\right)^{2n} = e^{-2}$

(b) $\lim_{x \rightarrow 0} (1 + 4x)^{1/x} = e^4$

2. Calculate the following.

(a) $\frac{d}{dx} (\ln(\tanh x)) = (\sinh x \cosh x)^{-1}$

(b) $\int \frac{z}{2z^2 + 8} dz = \frac{1}{4} \ln(2z^2 + 8) + C$

(c) $\int \frac{\tan(\ln x)}{x} dx = \ln |\sec(\ln x)| + C,$

(d) $\int \frac{dx}{x(1-x)} = \ln \left| \frac{x}{x-1} \right| + C,$

(e) $\frac{dy}{dx}, \quad y = \frac{(x^2 + 3)^{2/3}(3x + 2)^2}{\sqrt{x + 1}} \quad \frac{dy}{dx} = \left(\frac{4x}{3x^2 + 9} + \frac{6}{3x + 2} - \frac{1}{2x + 2} \right)$

(f) $\int \frac{e^x}{1 + e^{2x}} = \arctan(e^x) + C$

3. Experiments show that the rate of change of the atmospheric pressure $P(x)$ with altitude x is proportional to the pressure. Write down the resulting differential equation for $P(x)$, and solve it, assuming that the pressure at 6000 meters is half its value P_0 at sea level.

$$\frac{dP}{dx} = -kP, \quad k > 0; \quad P(x) = P_0 e^{-kx}, \quad k = (\ln 2)/6000$$

4. Section 6.5 #18

5,565 years ago

5. Section 6.3

40. $\ln |e^x - 1| + C$

46. $V = 2\pi \int_0^1 x e^{-x^2} dx = \pi \left(1 - \frac{1}{e}\right)$

6. Section 6.6

2. $y(x) = \frac{\frac{1}{3}x^3 - x}{x-1} + \frac{C}{x-1}$

4. $y(x) = \sin x + C \cos x$

7. Stewart wants to become a millionaire after 10 years by buying \$5,000 worth of a company's stock, which he wants to choose carefully. What must the sustained, annualized growth rate of the stock be in order to achieve his goal? Is Stewart being realistic?

70%

8. Newton's law of cooling states that the rate at which an object cools is proportional to the difference between the temperature $\theta(t)$ of the object and the constant ambient temperature T ,

$$\frac{d\theta}{dt} = -k(\theta - T),$$

where $k > 0$ is a constant depending on the object. A corpse is discovered at 2 pm, and its temperature is found to be 85°F , with the ambient air temperature being 68°F . Assuming $k = 0.5 \text{ hr}^{-1}$, find the time of death.

$\theta(t) = T + (\theta_0 - T)e^{-kt}$. The corpse is discovered at $t = 0$ (or 2 pm), and has a temperature $\theta_0 = \theta(0) = 85^\circ\text{F}$. Assume that t_d is the time of death and that $\theta(t_d) = \theta_d = 98.6^\circ\text{F}$. Thus

$$t_d = -\frac{1}{k} \ln\left(\frac{\theta_d - T}{\theta_0 - T}\right) = -1.176$$

or about 12:49 pm.

9. Section 6.8

45. $x^2 \left[\frac{xe^x}{1+e^{2x}} + 3 \arctan(e^x) \right]$

51. $\frac{3(1+\arcsin(x))^2}{\sqrt{1-x^2}}$

67. $\frac{1}{3} \arcsin\left(\frac{\sqrt{3}}{2}x\right) + C$

10. Section 6.9

25. $2 \tanh(x) \cosh(2x) + \sinh(2x) \operatorname{sech}^2(x)$

33. $\frac{1}{\sqrt{x^2-1} \cosh(x)}$

41. $2 \cosh(2z^{\frac{1}{4}}) + C$

45. $\frac{1}{4} [\ln(\sinh(x^2))]^2 + C$

11. Section 7.1

9. $2(4+z^2)^{\frac{3}{2}} + C$

19. $-\frac{1}{2} \cos(\ln(4x^2)) + C$

30. $\tan(e^x) + C$

12. Section 7.2

17. $\frac{2}{9} \left(e^{\frac{3}{2}} + 2 \right)$

23. $\frac{\pi}{2\sqrt{3}} + \ln(2)$

39. $z \ln^2(x) - 2z \ln(z) + 2z + C$

45. $\frac{\pi}{2} [\sin(\ln(x)) - \cos(\ln(x))] + C$

13. Section 7.3

5. $\frac{8}{15}$

15. $-\frac{1}{16}w - \frac{1}{32} \sin(2w) - \frac{1}{24} \sin^3(w) + C$