

Mathematics 1220 PRACTICE EXAM II Spring 2009

1. Calculate the following limits. Be sure to show all of your work.

(a)  $\lim_{n \rightarrow \infty} n (\sqrt[n]{n} - 1)$       (b)  $\lim_{x \rightarrow 0} (\cos x)^{\csc x}$       (c)  $\lim_{x \rightarrow +\infty} x^{25} e^{-x}$       (d)  $\lim_{x \rightarrow \infty} \frac{x e^{-x^2/2}}{e^{-x}}$

(e)  $\lim_{x \rightarrow -\infty} (e^{-x} - x)$       (f)  $\lim_{x \rightarrow 1^+} \frac{\int_1^x \sin t \, dt}{x - 1}$       (g)  $\lim_{x \rightarrow 0} \frac{\int_0^x (e^{t^2} - 1) \, dt}{x^3}$

(h)  $\lim_{x \rightarrow \infty} \frac{3x}{\ln(100x + e^x)}$       (i)  $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2}$

2. Calculate the following integrals. Be sure to show all of your work.

(a)  $\int_{-\pi}^{\pi} \sin mx \cos nx \, dx$       (b)  $\int_0^{2\pi} \sin mx \sin nx \, dx$ ,  $m = n$

(c)  $\int \tanh x \ln(\cosh x) \, dx$       (d)  $\int \frac{e^x dx}{e^{2x} + 2e^x + 5}$       (e)  $\int \frac{3x - 1}{x^2 - 4} \, dx$

(f)  $\int \cos(\ln x) \, dx$       (g)  $\int \frac{x \, dx}{2x^3 + 6x^2}$       (h)  $\int (t + 1)e^{-t^2 - 2t - 5} \, dt$

(i)  $\int \sin^3 x \, dx$       (j)  $\int x e^x \, dx$       (k)  $\int \frac{3x - 13}{x^2 + 3x - 10} \, dx$

3. Determine whether the following improper integrals converge or diverge. Be sure to justify your answer completely.

(a)  $\int_e^{\infty} \frac{dx}{x\sqrt{\ln x}}$       (b)  $\int_1^{\infty} \frac{\ln x}{\sqrt{x^3 + 2x + 1}} \, dx$       (c)  $\int_0^{\infty} e^{-x} \cos x \, dx$

(d)  $\int_{-\infty}^{\infty} \frac{e^{-x^2}}{x^2} \, dx$       (e)  $\int_0^{\infty} x^{16,000} e^{-x} \, dx$       (f)  $\int_0^1 \frac{e^{-x} \sqrt{1+x}}{\sqrt[3]{\sin x \tan x}} \, dx$

4. Determine whether the following infinite series converge or diverge. Be sure to justify your answers completely.

(a)  $\sum_{n=1}^{\infty} \left(\frac{3}{e}\right)^n$       (b)  $\sum_{k=2}^{\infty} \left(\frac{1}{k} - \frac{1}{k-1}\right)$       (c)  $\sum_{k=2}^{\infty} \frac{1}{k+2}$       (d)  $\sum_{n=1}^{\infty} \sqrt{n} \sin\left(\frac{1}{n}\right)$

(e)  $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$       (f)  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^\pi}$       (g)  $\sum_{n=1}^{\infty} \sqrt{1 - \cos\left(\frac{1}{n}\right)}$

5. A (zero dimensional) bull frog initially jumps a meter. On each successive jump, he can only go  $\frac{3}{4}$  of the distance of the previous jump. If he takes infinitely many jumps, how far does he travel?

6. Find the limits of the following sequences or prove they diverge. Be sure to justify your answer completely.

(a)  $\lim_{n \rightarrow \infty} (-1)^n \frac{n}{n+2}$

(b)  $\lim_{n \rightarrow \infty} \frac{\sqrt{n^2+4}}{2n+1}$

(c)  $\lim_{n \rightarrow \infty} \frac{\cos(2n)}{n^{1/2}}$

(d)  $\lim_{n \rightarrow \infty} \ln n - \ln(n+1)$