

Mathematics 1220 PRACTICE EXAM III Spring 2009

- Use the geometric series to find the Taylor series for $f(x) = \ln(1+x)$ around $x = 0$. Determine the radius of convergence of this series. Explain your result for the radius in terms of the singularity of f . Do the same for $f(x) = 1/(1+x)^2$.
- Use the geometric series to find the Taylor series for $f(x) = \frac{1}{4+x^2}$ around $x = 0$. Determine the radius of convergence for this series.
- Use the Taylor series around $x = 0$ for $\sin x$ and the trigonometric identity $2 \sin x \cos x = \sin(2x)$ to determine the Taylor series around $x = 0$ for $\sin^2 x$. Also determine the radius of convergence.
- Find the Taylor series for $\cosh x$ around $x = 0$ by using the series for e^x . What is its radius of convergence?
- Find the convergence set for the following power series. For (a), also analyze the type of convergence (or divergence) at the endpoints of the convergence set.

(a) $\sum_{n=1}^{\infty} \frac{(3x+1)^n}{n 2^n}$ (b) $\sum_{n=1}^{\infty} f_n x^n$, where $\{f_n\}$ is the Fibonacci sequence

- Find the following limits. Be sure to fully justify your answers.

(a) $\lim_{n \rightarrow \infty} e^{-n} \sin n$

(b) $\lim_{n \rightarrow \infty} (2n)^{1/2n}$

(c) $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right) \cos n\pi$

(d) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\exp\left(\frac{k}{n}\right)^2\right) \frac{1}{n}$

- A (zero dimensional) bull frog initially jumps a meter. On each successive jump, he can only go $\frac{3}{4}$ of the distance of the previous jump. If he takes infinitely many jumps, how far does he travel?
- Determine whether the following infinite series converge or diverge. If a series converges, determine whether the convergence is absolute or conditional. Be sure to justify your answers completely.

(a) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\tan^{-1} n}{1+n}$

(b) $\sum_{n=1}^{\infty} \sqrt{1 - \cos\left(\frac{1}{n}\right)}$

(c) $\sum_{n=1}^{\infty} \frac{n^{100}}{n!}$

(d) $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$

(e) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^\pi}$

(f) $\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$

- Use a power series to find the solution $y(x)$ to the differential equation

$$\frac{d^2 y}{dx^2} = -y$$

with initial conditions $y(0) = 0$ and $y'(0) = 1$.

10. Solve each of the following ODEs. If initial or other conditions are given, use them to find the constants in the general solution.

(a)

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - 2y = 0$$

(b)

$$2\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 2y = 0, \quad y(0) = 1, y'(0) = 2$$

(c)

$$2\frac{d^2y}{dt^2} + 5\frac{dy}{dt} = 0, \quad y(0) = 1, y'(0) = -1$$